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Dissipative Controllers for Nonlinear Multibody Flexible Space Systems

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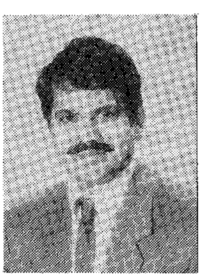
The problem of controlling a class of nonlinear multibody flexible space systems is considered. The system configuration consists of a flexible central body to which a number of flexible articulated appendages are attached, resulting in highly nonlinear dynamics. Assuming collocated actuators and sensors, global asymptotic stability of such systems is established using a nonlinear passivity-based control law. In addition, a special case where the central-body motion is small while the appendages can undergo unlimited motion, it is shown that the system, although highly nonlinear, can be stabilized by linear static and dynamic dissipative control laws. Furthermore, the static dissipative control law preserves stability despite actuator and sensor nonlinearities of certain types. In all cases, the stability does not depend on the knowledge of the model and hence is robust to modeling errors and uncertainties. The results are applicable to a broad class of systems, such as flexible multilink manipulators and multipayload space platforms. The stability proofs use the Lyapunov approach and exploit the inherent passivity of such systems.

I. Introduction

FLEXIBLE multibody space systems such as space platforms with multiple articulated payloads and space-based manipulators used for satellite assembly and servicing are characterized by significant flexibility in the structural members as well as joints. Examples of such systems include Earth-observing systems and Shuttle-based remote manipulator systems. Control systems design for such systems is a difficult problem because of the highly



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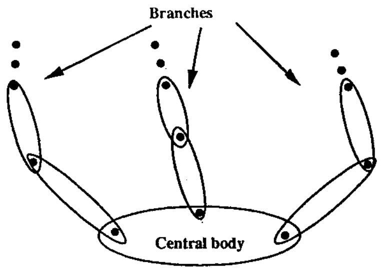

Fig. 1 Multibody system.

nonlinear dynamics, significant elastic motion with low inherent damping, and uncertainties in the mathematical model. The published literature contains a number of important stability results for certain subclasses of this problem, e.g., linear flexible structures, nonlinear multibody rigid structures, and recently, multibody flexible structures. Under certain conditions, the input–output maps for such systems can be shown to be "passive."[1] The Lyapunov and passivity approaches are used in Ref. 2 to demonstrate global asymptotic stability of linear flexible space structures (with no articulated appendages) for a class of dissipative compensators. The stability properties were shown to be robust to first-order actuator dynamics and certain actuator/sensor nonlinearities. Multibody rigid structures comprise another class of systems for which stability results have been advanced. Global asymptotic stability of terrestrial rigid manipulators has been established[3] employing proportional-plus-derivative control with gravity compensation, and the Lyapunov stability of nonlinear multilink flexible systems was addressed in Ref. 4. However, the global asymptotic stability of nonlinear, multilink, flexible space structures has not been addressed in the literature.

We consider a nonlinear rotational dynamic model of a multibody flexible spacecraft assumed to have a branched geometry; i.e., it has a central flexible body to which various flexible appendage bodies are attached (Fig. 1). A nonlinear mathematical model of a generic flexible multibody system is given in Sec. II. It is shown (Lemma A2 in the Appendix) that the system has the property of passivity that is pivotal to the proofs. Basic kinematic relations of the quaternion (a measure of attitude of the central body) are also given. Section III establishes the global asymptotic stability of the complete nonlinear system under a nonlinear control law based on quaternion feedback. A special case where the central-body attitude motion is small is addressed in Sec. IV. A numerical example is given in Sec. V. The control laws of Secs. III, IV.A, and IV.B were considered in Ref. 5 but are included here for completeness.

II. Mathematical Model

The class of systems considered consists of a branched configuration of flexible bodies as shown in Fig. 1. Each branch by itself could be a serial multibody structure. For the sake of simplicity and without loss of generality, we shall consider a spacecraft with only one such branch where each appendage body has one degree of freedom (hinge) with respect to the previous body in chain. However, the results obtained in this paper will also be applicable to the general case with multiple branches. Consider the spacecraft consisting of a central flexible body and a chain of $(k-3)$ flexible links. The central body has three rigid rotational degrees of freedom, and each link is connected by one rotational degree of freedom to the neighboring link. The Lagrangian for the system under consideration can be shown[5] to have the following form:

$$L = \tfrac{1}{2}[\dot{p}^T M(p)\dot{p} - q^T \tilde{K} q] \tag{1}$$

where $\dot{p} = \{\omega^T, \dot{\theta}^T, \dot{q}^T\}^T$; ω is the 3×1 inertial angular velocity vector (in body-fixed coordinates) for the central body; $\theta = (\theta_1, \theta_2, \ldots, \theta_{(k-3)})^T$, where θ_i denotes the joint angle for the ith joint expressed in body-fixed coordinates; q is an $(n-k)$ vector of flexible degrees of freedom (modal amplitudes); $M(p) = M^T(p) > 0$ is the configuration-dependent mass–inertia matrix; and $\tilde{K}$ is the symmetric positive-definite stiffness matrix related to the flexible degrees of freedom. Using the Lagrangian (1), the following equations of motion are obtained (the details of the derivation of the mathematical model can be found in Ref. 5):

$$M(p)\ddot{p} + C(p, \dot{p})\dot{p} + D\dot{p} + Kp = B^T u \tag{2}$$

where $\{p\} = \{\gamma^T, \theta^T, q^T\}^T$ and $\dot{\gamma} = \omega$. Here, $C(p, \dot{p})$ corresponds to Coriolis and centrifugal forces; D is the symmetric, positive-semidefinite damping matrix; $B = [I_{k\times k}\ 0_{k\times(n-k)}]$ is the control influence matrix; and u is the k-vector of applied torques. The first three components of u represent the attitude control torques (about the x, y, z axes) applied to the central body, whereas the remaining components are the torques applied at the $(k-3)$ joints. The terms K and D are symmetric, positive-semidefinite stiffness and damping matrices:

$$K = \begin{bmatrix} 0_{k\times k} & 0_{k\times(n-k)} \\ 0_{(n-k)\times k} & \tilde{K}_{(n-k)\times(n-k)} \end{bmatrix}, \quad D = \begin{bmatrix} 0_{k\times k} & 0_{k\times(n-k)} \\ 0_{(n-k)\times k} & \tilde{D}_{(n-k)\times(n-k)} \end{bmatrix} \tag{3}$$

where $\tilde{K}$ and $\tilde{D}$ are symmetric positive definite. The angular measurements for the central body are the three Euler angles (not the vector γ), whereas remaining angular measurements consist of the relative angles between adjoining bodies. The angular rate measurements are given by $y_r = (\omega^T, \dot{\theta}^T)^T = B\dot{p}$. One important inherent property of such systems that is crucial to the stability results is that the matrix $((1/2)\dot{M} - C)$ is skew symmetric. An outline of the proof of this property is given as Lemma A1 in the Appendix. (See Ref. 6 for a detailed proof.) Using this property, it is also shown in the Appendix (Lemma A2) that the input–output map from u to y_r is passive. Because of this passivity property, the system can be stabilized (in the input–output sense) by any strictly passive controller.[1] The simplest example of such a controller is constant (positive) gain rate feedback. However, the system would be stable only in the input–output sense and would not be asymptotically stable. In particular, the steady-state velocities will be zero, but the steady-state values of the central-body attitude and appendage angles can be arbitrary. In other words, the system would be only Lyapunov stable. Therefore, we shall develop control laws that utilize both position and rate measurements and yield asymptotic stability.

The central-body attitude (Euler angle) vector η is given by $E(\eta)\dot{\eta} = \omega$, where $E(\eta)$ is a 3×3 transformation matrix. The sensor outputs consist of three central-body Euler angles, the $(k-3)$ joint angles, and the angular rates, i.e., the sensors are collocated with the torque actuators. The sensor outputs are then given by

$$y_p = B\hat{p}, \qquad y_r = B\dot{p} \tag{4}$$

where $\hat{p} = (\eta^T, \theta^T, q^T)^T$ wherein η is the Euler angle vector for the central body. Here, $y_p = (\eta^T, \theta^T)^T$ and $y_r = (\omega^T, \dot{\theta}^T)^T$ are measured angular position and rate vectors, respectively. It is assumed that the body rate measurements ω are available via rate gyroscopes.

The orientation of a free-floating body can be minimally represented by a three-dimensional orientation vector. However, this representation is not unique. One minimal representation that is commonly used to represent the attitude is Euler angles. As stated previously, the 3×1 Euler angle vector η is given by $E(\eta)\dot{\eta} = \omega$, where $E(\eta)$ is a 3×3 transformation matrix. The problem with using this representation is that $E(\eta)$ becomes singular for certain values of η. The problem of singularity in three-parameter representation of attitude has been studied in detail in the literature. An effective way of overcoming the singularity problem is to use the quaternion formulation.[7]

The unit quaternion (also known as Euler parameter vector) α is defined as follows:

$$\alpha = \{\bar{\alpha}^T, \alpha_4\}^T, \quad \bar{\alpha} = \begin{bmatrix} \hat{\alpha}_1 \\ \hat{\alpha}_2 \\ \hat{\alpha}_3 \end{bmatrix} \sin\left(\frac{\phi}{2}\right), \quad \alpha_4 = \cos\left(\frac{\phi}{2}\right) \tag{5}$$

Here, $\hat{\alpha} = (\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3)^T$ is the unit vector along the eigenaxis of rotation and ϕ is the magnitude of rotation. The quaternion is also subjected to the norm constraint

$$\bar{\alpha}^T\bar{\alpha} + \alpha_4^2 = 1 \tag{6}$$

The quaternion obeys the following kinematic differential equations:

$$\dot{\bar{\alpha}} = \tfrac{1}{2}(\omega \times \bar{\alpha} + \alpha_4\omega) \qquad \dot{\alpha}_4 = -\tfrac{1}{2}\omega^T\bar{\alpha} \tag{7}$$

We shall use the quaternion representation for the central-body attitude. Euler angle measurements can be used to compute the quaternions.[7]

The open-loop system given by Eqs. (2), (7), and (8) has multiple equilibrium solutions: $(\bar{\alpha}_{ss}^T, \alpha_{4ss}, \theta_{ss}^T)^T$, where the subscript ss denotes the steady-state value (the steady-state value of q is zero). Defining $\beta = (\alpha_4 - 1)$ and denoting $\dot{p} = z$, Eqs. (2), (7), and (8) can be rewritten as

$$M\dot{z} + Cz + Dz + \tilde{K}q = B^T u \tag{8}$$

$$\begin{bmatrix} \dot{\theta} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0_{(n-3)\times 3} & I_{(n-3)\times(n-3)} \end{bmatrix} z \tag{9}$$

$$\dot{\bar{\alpha}} = \tfrac{1}{2}[\omega \times \bar{\alpha} + (\beta + 1)\omega] \tag{10}$$

$$\dot{\beta} = -\tfrac{1}{2}\omega^T\bar{\alpha} \tag{11}$$

In Eq. (8) the matrices M and C are functions of p and $(p, \dot{p})$, respectively. It should be noted that the first three elements of p associated with the orientation of the central body can be fully described by the unit quaternion. Hence, M and C are implicit functions of α, and therefore, the system represented by Eqs. (8–11) is time invariant and can be expressed in the state-space form as

$$\dot{x} = f(x, u), \qquad \tilde{y}_p = B\tilde{p}, \qquad y_r = B\dot{p} \tag{12}$$

where $x = (\bar{\alpha}^T, \beta, \theta^T, q^T, z^T)^T$ and $\tilde{p} = (\bar{\alpha}^T, \theta^T)^T$. Note that the dimension of x is $(2n + 1)$, which is one more than the dimension of the system in Eq. (2). However, one constraint [Eq. (6)] is now present. It can be easily verified from Eq. (7) that the constraint (6) is satisfied for all $t > 0$ if it is satisfied at $t = 0$.

III. Nonlinear Dissipative Control Law

Consider the dissipative control law u given by

$$u = -G_p\tilde{y}_p - G_r y_r \tag{13}$$

where matrices G_p and G_r are symmetric positive-definite $k \times k$ matrices and G_p is given by

$$G_p = \begin{bmatrix} \left(1 + \dfrac{(\beta+1)}{2}\right)G_{p1} & 0_{3\times(k-3)} \\ 0_{(k-3)\times 3} & G_{p2(k-3)\times(k-3)} \end{bmatrix} \tag{14}$$

Note that Eqs. (13) and (14) represent a nonlinear control law. If G_p and G_r satisfy certain conditions, this control law can be shown to render the time rate of change of the system's energy negative along all trajectories; i.e., it is a dissipative control law.

The closed-loop equilibrium solution can be obtained by equating all the derivatives to zero in Eqs. (2), (10), and (11). In particular, $\dot{p} = \ddot{p} = 0 \Rightarrow \omega = 0, \dot{\theta} = 0, \dot{q} = 0$, and

$$-B^T G_p\tilde{p} = \begin{bmatrix} -G_p\tilde{p} \\ 0_{(n-k)\times(n-k)} \end{bmatrix} = \begin{bmatrix} 0_{k\times 1} \\ \tilde{K}q \end{bmatrix} \tag{15}$$

Since $|\beta + 1| \leq 1$ [because of Eq. (6)], G_p is positive definite, and Eq. (15) implies $\tilde{p} = (\bar{\alpha}^T, \theta^T)^T = 0$ and $q = 0$. The equilibrium solution of Eq. (11) is $\beta = \beta_{ss}$ (constant), i.e., $\alpha_4 =$ constant, which implies [from Eq. (6)] that $\alpha_4 = \pm 1$. Thus there appear to be two closed-loop equilibrium points corresponding to $\alpha_4 = 1$ and $\alpha_4 = -1$ (all other state variables being zero). However, from Eq. (5), $\alpha_4 = 1 \Rightarrow \phi = 0$ and $\alpha_4 = -1 \Rightarrow \phi = 2\pi$, i.e., there is only one equilibrium point in the physical space.

One of the control objectives is to transfer the state of the system from one orientation (equilibrium) position to another orientation. Without loss of generality, the target orientation can be defined to be zero, and the initial orientation, given by $(\bar{\alpha}(0), \alpha_4(0), \theta(0))$, can always be defined in such a way that $|\phi(0)| \leq \pi, 0 \leq \alpha_4(0) \leq 1$ (corresponding to $|\phi| \leq \pi$), and $(\bar{\alpha}(0), \alpha_4(0))$ satisfy Eq. (6).

The following theorem establishes the global asymptotic stability of the physical equilibrium state of the system.

Theorem 1. Suppose $G_{p2(k-3)\times(k-3)}$ and $G_{r(k\times k)}$ are symmetric and positive definite and $G_{p1} = \mu I_3$, where $\mu > 0$. Then, the closed-loop system given by Eqs. (12) and (13) is globally asymptotically stable (GAS).

Proof. Consider the candidate Lyapunov function

$$V = \tfrac{1}{2}\dot{p}^T M(p)\dot{p} + \tfrac{1}{2}q^T\tilde{K}q + \tfrac{1}{2}\theta^T G_{p2}\theta$$
$$+ \tfrac{1}{2}\bar{\alpha}^T(G_{p1} + 2\mu I_3)\bar{\alpha} + \beta^2\mu \tag{16}$$

Here, V is clearly positive definite and radially unbounded with respect to a state vector $\{\bar{\alpha}^T, \beta, \theta^T, q^T, \dot{p}^T\}^T$ since $M(p)$, $\tilde{K}$, G_{p1}, and G_{p2} are positive-definite symmetric matrices. Note that the matrix $M(p)$, although configuration dependent, is uniformly bounded from below and above by the values that correspond to the minimum and maximum inertia configurations, respectively (i.e., there exist positive-definite matrices $\underline{M}$ and $\bar{M}$ such that $\underline{M} \leq M \leq \bar{M}$). Taking the time derivative of V, we have

$$\dot{V} = \dot{p}^T M\ddot{p} + \tfrac{1}{2}\dot{p}^T\dot{M}\dot{p} + \dot{q}^T\tilde{K}q + \dot{\theta}^T G_{p2}\theta$$
$$+ \dot{\bar{\alpha}}^T(G_{p1} + 2\mu I_3)\bar{\alpha} + 2\beta\dot{\beta}\mu \tag{17}$$

Using Eqs. (2), (10), (11), and (14), we get

$$\dot{V} = \dot{p}^T B^T u + \dot{p}^T\left(\tfrac{1}{2}\dot{M} - C\right)\dot{p} - \dot{p}^T D\dot{p} - \dot{p}^T Kp + \dot{q}^T\tilde{K}q$$
$$+ \dot{\theta}^T G_{p2}\theta + \tfrac{1}{2}(\Omega\bar{\alpha})^T G_{p1}\bar{\alpha} + \tfrac{1}{2}(\beta + 1)\omega^T G_{p1}\bar{\alpha} + \mu\omega^T\bar{\alpha} \tag{18}$$

where $\Omega = (\omega\times)$ denotes the skew-symmetric cross product matrix, i.e., $\omega \times x = \Omega x$. Substituting for u and noting that $\dot{p}^T Kp = \dot{q}^T\tilde{K}q$, $(\Omega\bar{\alpha})^T G_{p1}\bar{\alpha} = 0$, and using Lemma A1, we obtain

$$\dot{V} = -\dot{p}^T\left(D + B^T G_r B\right)\dot{p} - (B\dot{p})^T G_p\tilde{p}$$
$$+ \tfrac{1}{2}(\beta + 1)\omega^T G_{p1}\bar{\alpha} + \mu\omega^T\bar{\alpha} + \dot{\theta}^T G_{p2}\theta \tag{19}$$

Note that $(B\dot{p})^T G_p\tilde{p} = (1/2)(\beta + 1)\omega^T G_{p1}\bar{\alpha} + \mu\omega^T\bar{\alpha} + \dot{\theta}^T G_{p2}\theta$. After cancellations, we get

$$\dot{V} = -\dot{p}^T\left(D + B^T G_r B\right)\dot{p} \tag{20}$$

Since $D + B^T G_r B$ is a positive-definite symmetric matrix, $\dot{V} \leq 0$, i.e., $\dot{V}$ is negative semidefinite, and $\dot{V} = 0 \Rightarrow \dot{p} = 0 \Rightarrow \ddot{p} = 0$. Substituting in the closed-loop equation we get Eq. (15). As shown previously, Eq. (15) $\Rightarrow \tilde{p} = 0, q = 0$, i.e., $\bar{\alpha} = 0, \theta = 0$, and $\alpha_4 = \pm 1$ (or $\beta = 0, -2$). Consistent with the previous discussion, these values correspond to two equilibrium points representing the same physical equilibrium state.

It can be easily verified, from Eq. (16), that any small perturbation ϵ in α_4 from the equilibrium point corresponding to $\alpha_4 = -1$ will cause a decrease in the value of V(ϵ has to be greater than 0 because $|\alpha_4| \leq 1$). Thus, in the mathematical sense, $\alpha_4 = -1$ corresponds to an isolated equilibrium point such that $\dot{V} = 0$ at that point and $\dot{V} < 0$ in a neighborhood of that point, i.e., $\alpha_4 = -1$ is a "repeller," not an "attractor." It has been already shown that $\dot{V}$ is negative along all trajectories in the state space except at the two equilibrium points. That is, if the system's initial condition lies anywhere in the state space except at the equilibrium point corresponding to $\alpha_4 = -1$, then the system will asymptotically approach the origin, i.e., $x = 0$; and if the system is at the equilibrium point corresponding to $\alpha_4 = -1$ at $t = 0$, then it will stay there for all $t > 0$. However, this is the same equilibrium point in the physical space; hence it can be concluded by LaSalle's invariance theorem that the system is GAS. □

IV. Systems in Attitude Hold Configuration

Consider an important special case where the central-body attitude motion is small. This can occur in many realistic situations. For example, in the case of a space-station-based or Shuttle-based manipulator, the inertia of the base (central body) is much larger than that of any manipulator link or payload. In such cases the rotational motion of the base can be assumed to be in the linear region, although the payloads (or links) attached to it can undergo large rotational and translational motions and nonlinear dynamic loading due to Coriolis and centripetal accelerations. That is, the system model is still highly nonlinear. For this case, since the attitude motion of the central body is small, the singularity problem discussed in Sec. II does not arise, and the quaternion formulation is not necessary. As a result, γ approximately equals the Euler angle vector η and can be used to represent the central-body attitude. Here, γ is given by $\dot{\gamma} = \omega$. The equations of motion (2) can now be expressed in the state-space form simply as

$$\dot{\bar{x}} = \begin{bmatrix} 0 & I \\ -M^{-1}(p)K & -M^{-1}(p)(C(p,\dot{p})+D) \end{bmatrix}\bar{x} + \begin{bmatrix} 0 \\ M^{-1}(p)B^T \end{bmatrix} u \tag{21}$$

where $\bar{x} = \{p^T, \dot{p}^T\}^T$, $p = \{\gamma^T, \theta^T, q^T\}^T$. Note that Eq. (21) still represents a highly nonlinear dynamic system. However, it is shown next that it can be stabilized by linear dissipative controllers.

A. Stability with Static Dissipative Controllers

The static dissipative control law u is given by

$$u = -\bar{G}_p y_p - G_r y_r \tag{22}$$

where $\bar{G}_p$ is a symmetric positive-definite $k \times k$ matrix

$$y_p = Bp = (\gamma^T, \theta^T)^T, \qquad y_r = B\dot{p} \tag{23}$$

where y_p and y_r are measured angular position and rate vectors.

Theorem 2. Suppose $\bar{G}_{p,k\times k}$ and $G_{r,k\times k}$ are symmetric and positive definite. Then, the closed-loop system given by Eqs. (21–23) is GAS.

Proof. Consider the candidate Lyapunov function

$$V = \tfrac{1}{2}\dot{p}^T M(p)\dot{p} + \tfrac{1}{2}p^T\left(K + B^T\bar{G}_p B\right)p \tag{24}$$

Here, V is clearly positive definite since $M(p)$ and $K + B^T\bar{G}_p B$ are positive-definite symmetric matrices. Taking the time derivative, letting $\bar{K} = (K + B^T\bar{G}_p B)$, and simplifying, we get

$$\dot{V} = \dot{p}^T\left(\tfrac{1}{2}\dot{M} - C\right)\dot{p} - \dot{p}^T\bar{K}p + \dot{p}^T\bar{K}p - \dot{p}^T\left(D + B^T G_r B\right)\dot{p} \tag{25}$$

Again, using Lemma A1, we get, $\dot{p}^T((1/2)\dot{M} - C)\dot{p} = 0$, and after some cancellations, we obtain

$$\dot{V} = -\dot{p}^T\left(D + B^T G_r B\right)\dot{p} \tag{26}$$

Since $D + B^T G_r B$ is the positive-definite symmetric matrix, $\dot{V} \le 0$, i.e., $\dot{V}$ is negative semidefinite in p and $\dot{p}$ and $\dot{V} = 0 \Rightarrow \dot{p} = 0 \Rightarrow \ddot{p} = 0$. Substituting for u from Eq. (22) into Eq. (2), we get the closed-loop steady-state equation

$$\left(K + B^T\bar{G}_p B\right)p = 0 \Rightarrow p = 0 \tag{27}$$

Thus, $\dot{V}$ is not zero along any trajectories; then, by LaSalle's theorem, the system is GAS. □

The significance of the two results presented in Theorems 1 and 2 is that nonlinear multibody systems belonging to these classes can be stabilized with the dissipative control laws given. The control laws do not require any knowledge of the system parameters or model order and depend only on the inherent input–output property of the system, namely passivity, which is a result of actuator/sensor collocation. Therefore, the control laws given are robust to modeling errors and parametric uncertainties. In the case of manipulators, this means that one can accomplish any terminal angular position from any initial position with guaranteed asymptotic stability.

B. Robustness to Actuator/Sensor Nonlinearities

Theorem 2 proves global asymptotic stability for systems in the central-body attitude-hold configuration. It assumes linear actuators and sensors. In practice, however, the actuators and sensors have nonlinearities. The following theorem extends the results of Ref. 2 to the case of nonlinear flexible multibody systems. That is, the robust stability property of the dissipative controller is proved to hold in the presence of a wide class of actuator/sensor nonlinearities (such as saturation) as defined below.

Definition. A function $\psi(\nu)$ is said to belong to the $(0,\infty)$ sector if $\psi(0) = 0$ and $\nu\psi(\nu) > 0$ for $\nu \neq 0$: ψ is said to belong to the $[0,\infty)$ sector if $\nu\psi(\nu) \ge 0$.

Let $\psi_{ai}(\cdot)$, $\psi_{pi}(\cdot)$, and $\psi_{ri}(\cdot)$ denote the nonlinearities in the ith actuator, position sensor, and rate sensor channels, respectively. Assuming $\bar{G}_p$ and G_r are diagonal with elements $\bar{G}_{pi}$ and G_{ri}, respectively, the actual input is given by

$$u_i = \psi_{ai}[-\bar{G}_{pi}\psi_{pi}(y_{pi}) - G_{ri}\psi_{ri}(y_{ri})] \qquad i = 1, 2, \ldots, k \tag{28}$$

We assume that ψ_{pi}, ψ_{ai}, and ψ_{ri} $(i = 1, 2, \ldots, k)$ are continuous single-valued functions $\boldsymbol{R} \to \boldsymbol{R}$. The following theorem gives sufficient conditions for stability.

Theorem 3. Consider the closed-loop system given by Eqs. (21), (23), and (28), where $\bar{G}_p$ and G_r are diagonal with positive entries. Suppose ψ_{ai}, ψ_{pi}, and ψ_{ri} $(i = 1, 2, \ldots, k)$ are single-valued, time-invariant continuous functions belonging to the $(0,\infty)$ sector and ψ_{ai} are monotonically nondecreasing. Under these conditions, the closed-loop system is GAS.

Proof. (The proof closely follows Ref. 2.) Let $\varphi = -y_p$. Define

$$\bar{\psi}_{pi}(\nu) = -\psi_{pi}(-\nu) \tag{29}$$

$$\bar{\psi}_{ri}(\nu) = -\psi_{ri}(-\nu) \tag{30}$$

If $\psi_{pi}, \psi_{ri} \in (0,\infty)$ sector, then $\bar{\psi}_{pi}, \bar{\psi}_{ri}$ also belong to the same sector. Now, consider the following Luré–Postnikov Lyapunov function:

$$V = \frac{1}{2}\dot{p}^T M(p)\dot{p} + \frac{1}{2}q^T\tilde{K}q + \sum_{i=1}^{k}\int_0^{\varphi_i}\psi_{ai}[\bar{G}_{pi}\bar{\psi}_{pi}(\nu)]\,\mathrm{d}\nu \tag{31}$$

where $\tilde{K}$ is the symmetric positive-definite part of K. Taking the time derivative and using (2),

$$\dot{V} = \dot{p}^T(B^T u - C\dot{p} - D\dot{p} - Kp) + \frac{1}{2}\dot{p}^T\dot{M}\dot{p} + \sum_{i=1}^{k}\dot{\varphi}_i\psi_{ai}[\bar{G}_{pi}\bar{\psi}_{pi}(\varphi_i)] + \dot{q}^T\tilde{K}q \tag{32}$$

Upon several cancellations and using Lemma A1,

$$\dot{V} = -\sum_{i=1}^{k}u_i\dot{\varphi}_i - \dot{q}^T\tilde{D}\dot{q} + \sum_{i=1}^{k}\dot{\varphi}_i\psi_{ai}[\bar{G}_{pi}\bar{\psi}_{pi}(\varphi_i)] \tag{33}$$

where matrix $\tilde{D}$ is the positive-definite part of D:

$$\dot{V} = -\dot{q}^T\tilde{D}\dot{q} - \sum_{i=1}^{k}\dot{\varphi}_i\{\psi_{ai}[G_{ri}\bar{\psi}_{ri}(\dot{\varphi}_i) + \bar{G}_{pi}\bar{\psi}_{pi}(\varphi_i)] - \psi_{ai}[\bar{G}_{pi}\bar{\psi}_{pi}(\varphi_i)]\} \tag{34}$$

Since the ψ_{ai} are monotonically nondecreasing and ψ_{ri} belong to the $(0,\infty)$ sector, $\dot{V} \le 0$, and it can be concluded that the system is at least Lyapunov stable. Now we will prove that in fact the system is GAS. First, let us consider a special case when the ψ_{ai} are monotonically increasing. Then $\dot{V} \le -\dot{q}^T\tilde{D}\dot{q}$, and $\dot{V} = 0$ only when $\dot{q} = 0$ and $\dot{\varphi} = 0$, which implies $(\dot{\gamma}^T, \dot{\theta}^T)^T = 0 \Rightarrow \dot{p} = 0 \Rightarrow \ddot{p} = 0$. Substituting in the closed-loop equation,

$$Kp = B^T \psi_a[-\bar{G}_p \psi_p(y_p)] \tag{35}$$

$$\begin{bmatrix} 0 \\ \tilde{K}q \end{bmatrix} = \begin{bmatrix} \psi_a[-\bar{G}_p \psi_p(y_p)] \\ 0 \end{bmatrix} \tag{36}$$

$$\Rightarrow \psi_a[-\bar{G}_p \psi_p(y_p)] = 0 \qquad \text{and} \qquad q = 0$$

If the ψ_{pi} belong to the $(0, \infty)$ sector, $\psi_{ai}(\nu) = \psi_{pi}(\nu) = 0$ only when $\nu = 0$. Therefore, $y_p = 0$, i.e., $\theta = 0$ and $\gamma = 0$. Thus, $\dot{V} = 0$ only at the origin, and the system is GAS.

In the case when actuator nonlinearities are of the monotonically nondecreasing type (such as saturation nonlinearity), $\dot{V}$ can be 0 even if $\dot{\varphi} \neq 0$. However, we will show that every system trajectory along which $\dot{V} \equiv 0$ has to go to the origin asymptotically. When $\dot{\varphi} \neq 0$, $\dot{V} \equiv 0$ only when all actuators are saturated. Then, from the equations of motion, this implies that system trajectories will become unbounded, which is not possible since we have already proved that the system is Lyapunov stable. Hence, $\dot{V}$ cannot be identically zero along the system trajectories and the system is GAS. □

For the case considered in Sec. II, where the central-body motion is not in the linear range, the robust stability results in the presence of actuator/sensor nonlinearities cannot be easily extended since the stabilizing control law (13) is nonlinear.

The next section extends the robust stability results of Sec. IV.A to a class of more versatile controllers, namely, dynamic dissipative controllers. The advantages of using dynamic dissipative controllers include higher performance, more design freedom, and better noise attenuation.

C. Stability with Dynamic Dissipative Controllers

In order to obtain better performance while still retaining guaranteed robustness to unmodeled dynamics and parameter uncertainties, we consider a class of dynamic dissipative controllers (DDCs). Such compensators were suggested in the past for controlling only the elastic motion[8,9] of linear flexible space structures with no articulated appendages (i.e., single-body structures). The results were extended in Ref. 10 to additionally include the rigid-body modes. The results to be presented in this section essentially extend and generalize the results of Ref. 10 to the nonlinear multibody case. In particular, it is shown that nonlinear multibody space structures in the attitude-hold configuration can be stabilized by linear DDCs. The methods of proof are similar to those in Ref. 10, which addressed only linear single-body spacecraft.

Mathematical Preliminaries

Definition. A rational matrix-valued function $T(s)$ of the complex variable s is said to be positive real if all of its elements are analytic in $Re[s] > 0$, and $T(j\omega) + T^*(j\omega) \geq 0$ for $\omega \in (-\infty, \infty)$, where the asterisk denotes the complex conjugate transpose.

Suppose (A, B, C, D) is an nth-order minimal realization of $T(s)$. From Ref. 11, a necessary and sufficient condition for $T(s)$ to be positive real is that there exists an $n \times n$ symmetric positive-definite matrix P and matrices W and L such that

$$A^T P + PA = -LL^T, \quad C = B^T P + W^T L, \quad W^T W = D + D^T \tag{37}$$

This result is also generally known in the literature as the Kalman–Yakubovich lemma. A stronger concept along these lines is strictly positive-real (SPR) systems. However, there are several definitions of SPR systems (see Ref. 12). The concept of weakly SPR[12] appears to be the least restrictive definition of SPR. Nevertheless, all the definitions of SPR seem to require the system to have all poles in the open left-half plane. Herein we define marginally strictly positive-real[13] systems as follows:

Definition. A rational matrix-valued function $T(s)$ of the complex variable s is said to be marginally strictly positive real (MSPR) if $T(s)$ is positive real and $T(j\omega) + T^*(j\omega) > 0$ for $\omega \in (-\infty, \infty)$.

The obvious difference between the above definition and the definition of positive-real systems is that the $\geq$ has been replaced by strict inequality. The difference between the MSPR and weak SPR of Ref. 12 is that the latter definition requires the system to have poles in the open left-half plane, whereas the former definition allows poles on the $j\omega$ axis. It was proved in Ref. 13 that a linear positive-real system can be robustly stabilized by any MSPR control law. It is proved next that the nonlinear multibody flexible system (21) can also be robustly stabilized by an MSPR control law.

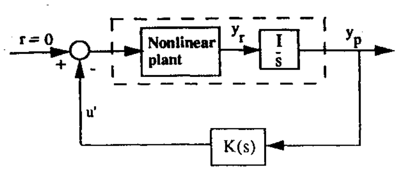

Fig. 2 Feedback configuration.

Stability Results

Consider the system given by Eq. (21) with the sensor outputs given by Eq. (23). Suppose a controller $\mathcal{K}(s)$, with k inputs and k outputs, is represented by the minimal realization

$$\dot{x}_c = A_c x_c + B_c u_c \tag{38}$$

$$y_c = C_c x_c + D_c u_c \tag{39}$$

where x_c is the n_c-dimensional state vector and (A_c, B_c, C_c, D_c) is a minimal realization of $\mathcal{K}(s)$.

Define

$$\dot{v} = y_c \tag{40}$$

$$x_z = \left(x_c^T, v^T\right)^T \tag{41}$$

$$y_z = v \tag{42}$$

Equations (39–42) can be combined as

$$\dot{x}_z = A_z x_z + B_z u_c \tag{43}$$

$$y_z = C_z x_z \tag{44}$$

where

$$A_z = \begin{bmatrix} A_c & 0 \\ C_c & 0 \end{bmatrix}, \qquad B_z = \begin{bmatrix} B_c \\ D_c \end{bmatrix}, \qquad C_z = [0 \quad I_k] \tag{45}$$

The closed-loop system is shown in Fig. 2. Here, $\mathcal{K}(s)$ is said to stabilize the nonlinear plant if the closed-loop system is GAS [with $\mathcal{K}(s)$ represented by its minimal realization].

Theorem 4. Consider the nonlinear plant (21) with y_p as the output. Suppose

1) A_c is strictly Hurwitz.

2) There exists an $(n_c + k) \times (n_c + k)$ matrix $P_z = P_z^T > 0$ such that

$$A_z^T P_z + P_z A_z = -Q_z \equiv -\text{diag}\left(L_c^T L_c, 0_k\right) \tag{46}$$

where L_c is the $k \times n_c$ matrix such that (L_c, A_c) is observable and $L_c(sI - A_c)^{-1} B_c$ has no transmission zeros in $Re[s] \geq 0$.

3) Assume

$$C_z = B_z^T P_z \tag{47}$$

4) $\mathcal{K}(s) = C_c(sI - A_c)^{-1} B_c + D_c$ has no transmission zeros at the origin.

Then $\mathcal{K}(s)$ stabilizes the nonlinear plant.

Proof. Let us first consider the system shown in Fig. 3a. The nonlinear plant is given by Eq. (2), and its state vector is taken to be $(q^T, \dot{p}^T)^T$; i.e., $(\gamma^T, \theta^T)^T$ is not included in the state vector. Now consider the Lyapunov function

$$V = \tfrac{1}{2}\dot{p}^T M(p)\dot{p} + \tfrac{1}{2}q^T \tilde{K} q + \tfrac{1}{2}x_z^T P_z x_z \tag{48}$$

where $\tilde{K}$ is the symmetric positive-definite part of K (i.e., the part associated with nonzero stiffness). Note that V is positive definite in the state vector $(q^T, \dot{p}^T)^T$ since the mass–inertia matrix $M(p)$,

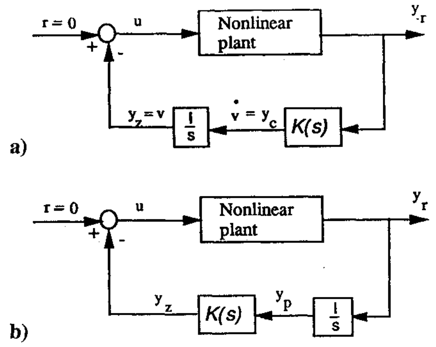

Fig. 3 Rearrangement of feedback loop.

as stated previously, is symmetric positive definite and uniformly bounded from below and above. Then

$$\dot{V} = \dot{p}^T M(p)\ddot{p} + \tfrac{1}{2}\dot{p}^T \dot{M}\dot{p} + \dot{q}^T \tilde{K} q + \tfrac{1}{2}\left(\dot{x}_z^T P_z x_z + x_z^T P_z \dot{x}_z\right) \quad (49)$$

After substituting for $M(p)\ddot{p}$ using Eq. (2) and for $\dot{x}_z$ using Eq. (43), Eq. (49) becomes

$$\dot{V} = \dot{p}^T B^T u - \dot{q}^T \tilde{D}\dot{q} + \dot{p}^T\left(\tfrac{1}{2}\dot{M} - C\right)\dot{p} - \dot{p}^T K p + \dot{q}^T \tilde{K} q$$
$$+ \tfrac{1}{2}\left[\left(x_z^T A_z^T + u_c^T B_z^T\right)P_z x_z + x_z^T P_z (A_z x_z + B_z u_c)\right] \quad (50)$$

Now using Lemma A1, the matrix $(1/2)\dot{M} - C$ is skew symmetric, and we get

$$\dot{V} = \dot{p}^T B^T u - \dot{q}^T \tilde{D}\dot{q} + \tfrac{1}{2}x_z^T\left(A_z^T P_z + P_z A_z\right)x_z$$
$$+ \tfrac{1}{2}u_c^T\left(B_z^T P_z\right)x_z + \tfrac{1}{2}x_z^T (P_z B_z) u_c \quad (51)$$

$$\dot{V} = -\dot{q}^T \tilde{D}\dot{q} + \dot{p}^T B^T u - \tfrac{1}{2}x_z^T Q_z x_z + x_z^T C_z^T u_c \quad (52)$$

Using Eqs. (46) and (47) and noting (Fig. 3a) that $u = -y_z = -C_z x_z$ and $B\dot{p} = y_r = u_c$,

$$\dot{V} = -\dot{q}^T \tilde{D}\dot{q} - \tfrac{1}{2}x_z^T Q_z x_z - u_c^T y_z + y_z^T u_c \quad (53)$$

$$\dot{V} = -\dot{q}^T \tilde{D}\dot{q} - \tfrac{1}{2}x_z^T Q_z x_z \quad (54)$$

Since $\tilde{D}$ is positive definite, it follows that $\dot{V} \le 0$; i.e., $\dot{V}$ is negative semidefinite, and the system is Lyapunov stable. Now, $\dot{V} = 0$ only if $\dot{q} = 0$ and $L_c x_c = 0$. Therefore, either $y_r = 0$ or y_r consists only of terms such as $vt^l e^{z_0 t}$, where v is a constant vector and z_0 is a transmission zero of (A_c, B_c, L_c). Since (A_c, B_c, L_c) has no transmission zeros in $Re(s) \ge 0$, this requires that $y_r \to 0$ exponentially. Since (A_c, B_c, L_c) is minimal and stable, $x_c \to 0$ exponentially. But $y_r \to 0 \Rightarrow \dot{\theta} \to 0$ and $\omega \to 0 \Rightarrow \dot{p} \to 0$; then this implies that $\ddot{p} \to 0$. Substituting in Eq. (2), we get $\gamma \to \gamma_{ss}$, $\theta \to \theta_{ss}$, $q \to 0$, and $u \to 0$, where γ_{ss} and θ_{ss} are some steady-state values of γ and θ, respectively.

Now consider the configuration shown in Fig. 3b, which is realized by applying the following similarity transformation to the system in Eq. (43):

$$\hat{T} = \begin{bmatrix} I_k & 0 & 0 \\ 0 & A_c & B_c \\ 0 & C_c & D_c \end{bmatrix} \quad (55)$$

Clearly, $\hat{T}$ is nonsingular if and only if $\mathcal{K}(s)$ has no transmission zeros at the origin. The transformed system has controller state equations

$$\dot{x}_c = A_c x_c + B_c y_p \quad (56)$$

$$u = -y_z = -(C_c x_c + D_c y_p) \quad (57)$$

where $y_p = (\gamma^T, \theta^T)^T$. Since transformation $\hat{T}$ is linear and nonsingular, the transformed system is Lyapunov stable. Now it will be shown that the system is, in fact, asymptotically stable.

Referring to Fig. 3b, we have shown that the output y_p tends to some steady-state value $\bar{y}_p = (\gamma_{ss}^T, \theta_{ss}^T)^T$. Since $\mathcal{K}(s)$ has no zeros at the origin and is stable, its output $y_z (= -u)$ will also tend to some steady-state $\bar{y}_z$. Consequently, if $\bar{y}_p \neq 0$, the control input u will tend to a constant value $\bar{u} \neq 0$. However, this contradicts the previously proven fact that $u \to 0$. Therefore, $y_p \to 0$ and $x_c \to 0$ [because $\mathcal{K}(s)$ is stable]. This proves (using LaSalle's invariance theorem) that the system is asymptotically stable. Since V is radially unbounded, the system is GAS. □

Since no assumptions were made regarding the model order as well as the knowledge of the parametric values, the stability is robust to modeling errors and parametric uncertainties. The robustness is a direct consequence of the passivity of the system, which results from actuator/sensor collocation.

Remark 1. In Theorem 4, if Eq. (46) holds with a negative-definite matrix Q_c replacing $L_c^T L_c$, then the closed-loop system is GAS. In this case the observability and minimum phase conditions in 2 are not needed.

Remark 2. The controller $\mathcal{K}(s)$ stabilizes the complete plant; i.e., the system consisting of the rigid modes, the elastic modes, and the compensator state vector (x_c) is GAS. The global asymptotic stability is guaranteed regardless of the number of modes in the model or parameter uncertainties. The order of the controller can be chosen to be any number $\ge k$. In other words, this result enables the design of a controller of essentially any desired order, which robustly stabilizes the plant. A procedure for designing $\mathcal{K}$ is to choose $Q_z = \mathrm{diag}(Q_c, 0_k)$, where $Q_c = Q_c^T > 0$, and to choose a stable A_c and matrices B_c and C_c so that Eqs. (46) and (47) have a positive-definite solution P_z. In addition, D_c must be such that $\mathcal{K}(s)$ has no transmission zeros at the origin, i.e., $\det[D_c - C_c A_c^{-1} B_c] \neq 0$. Because of the large number of free parameters $(A_c, B_c, C_c, D_c, L_c)$, it is generally not straightforward to use Theorem 4 to obtain the compensator. Another method is to use an s-domain sufficient condition, given below.

Theorem 5. The closed-loop system given by Eqs. (21), (56), and (57) is GAS if $\mathcal{K}(s)$ has no transmission zeros at $s = 0$ and $\mathcal{K}(s)/s$ is MSPR.

Proof. The proof can be obtained by a slight modification of the results of Ref. 13 to show that the above implies the conditions of Theorem 4.

The condition that $\mathcal{K}(s)/s$ be MSPR is sometimes much easier to check than the conditions of Theorem 1. For example, let $\mathcal{K}(s) = \mathrm{diag}[\mathcal{K}_1(s), \ldots, \mathcal{K}_k(s)]$, where

$$\mathcal{K}_i(s) = k_i \frac{s^2 + \beta_{1i}s + \beta_{0i}}{s^2 + \alpha_{1i}s + \alpha_{0i}} \quad (58)$$

It is straightforward to show that $\mathcal{K}(s)/s$ is MSPR if and only if (for $i = 1, \ldots, k$), $k_i, \alpha_{0i}, \alpha_{1i}, \beta_{0i}, \beta_{1i}$ are positive and

$$\alpha_{1i} - \beta_{1i} > 0, \qquad \alpha_{1i}\beta_{0i} - \alpha_{0i}\beta_{1i} > 0 \quad (59)$$

For higher order $\mathcal{K}_i$, the conditions on the polynomial coefficients are harder to obtain. One systematic procedure for obtaining such conditions for higher order controllers is the application of Sturm's theorem.[14] Symbolic manipulation codes can then be used to derive explicit inequalities. The controller design problem can be subsequently posed as a constrained optimization problem that minimizes a given performance function. For the case of fully populated $\mathcal{K}(s)$, however, there appear to be no straightforward methods and it remains an area of future research.

The following results, which address the cases with static dissipative controllers when the actuators have first- and second-order dynamics, are an immediate consequence of Theorem 5 and are stated without proof.

Corollary 1. For the static dissipative controller [Eq. (22)], suppose G_p and G_r are diagonal with positive entries (denoted by subscript i), and actuators represented by the transfer function $G_{Ai}(s) = k_i/(s + a_i)$ are present in the ith control channel. Then the closed-loop system is GAS if $G_{ri} > G_{pi}/a_i$ (for $i = 1, \ldots, k$).

Corollary 2. Suppose the static dissipative controller also includes the feedback of the acceleration y_a, that is,

$$u = -G_p y_p - G_r y_r - G_a y_a$$

where G_p, G_r, and G_a are diagonal with positive entries. Suppose the actuator dynamics for the ith input channel are given by $G_{Ai}(s) = k_i/(s^2 + \mu_i s + \nu_i)$, with k_i, μ_i, ν_i positive. Then the closed-loop system is asymptotically stable if

$$\frac{G_{ri}}{G_{ai}} \leq \mu_i < \frac{G_{ri}}{G_{pi}} \qquad i = 1, \ldots, k$$

Realization of $\mathcal{K}$ as a Strictly Proper Controller

The controller $\mathcal{K}(s)$ [Eqs. (56) and (57)] is not strictly proper because of the direct transmission term D_c. From a practical viewpoint, it is sometimes desirable to have a strictly proper controller because it attenuates sensor noise as well as high-frequency disturbances. Furthermore, the most common types of controllers, which include the linear quadratic Gaussian (LQG) as well as the observer/pole placement controllers, are strictly proper (they have a first-order rolloff). In addition, the realization in Eqs. (56) and (57) does not utilize the rate measurement y_r. The following result states that K can be realized as a strictly proper controller wherein both y_p and y_r are utilized.

Theorem 6. The nonlinear plant with y_p and y_r as outputs is stabilized by the controller $\mathcal{K}'$ given by

$$\dot{\bar{x}}_c = A_c\bar{x}_c + [B_c - A_c L \quad L]\begin{bmatrix} y_p \\ y_r \end{bmatrix} \tag{60}$$

$$y_c = C_c\bar{x}_c \tag{61}$$

where C_c is assumed to be of full rank and an $n_c \times k (n_c \geq k)$ matrix L is a solution of

$$D_c - C_c L = 0 \tag{62}$$

Proof. Consider the controller realization (56) and (57). Let

$$\bar{x}_c = x_c + L y_p \tag{63}$$

where L is an $n_c \times k$ matrix. Differentiating Eq. (63), using Eqs. (56) and (57), and replacing $\dot{y}_p$ by y_r, we get Eq. (60), and

$$y_c = C_c\bar{x}_c + (D_c - C_c L)y_p \tag{64}$$

If L is chosen to satisfy Eq. (62), we get the strictly proper controller given by Eqs. (60) and (61). Equation (62) represents k^2 equations in kn_c unknowns. If $k < n_c$ (i.e., the compensator order is greater than the number of plant inputs), and C_c is of full rank, there are many possible solutions for L. The solution that minimizes the Frobenius norm of L is

$$L = C_c^T\left(C_c C_c^T\right)^{-1} D_c \tag{65}$$

If $k = n_c$, Eq. (65) gives the unique solution $L = C_c^{-1} D_c$.

V. Numerical Example: Two-Link Flexible Space Robot

A numerical example is given to demonstrate the result of Sec. III. The example system consists of a conceptual nonlinear model of a spacecraft (Fig. 4) consisting of a central body with two flexible articulated appendages. The stability result obtained for the nonlinear dissipative control law given in Sec. III is verified by simulation of this system, which resembles a flexible space manipulator. The central body is a solid cylinder with 1.0 m diameter and 2 m height,

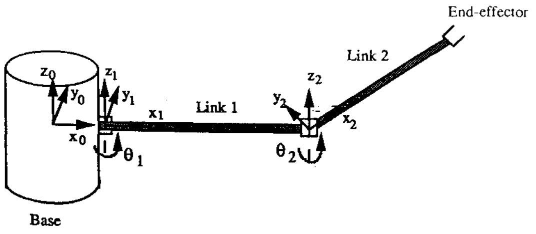

Fig. 4 Flexible space robot.

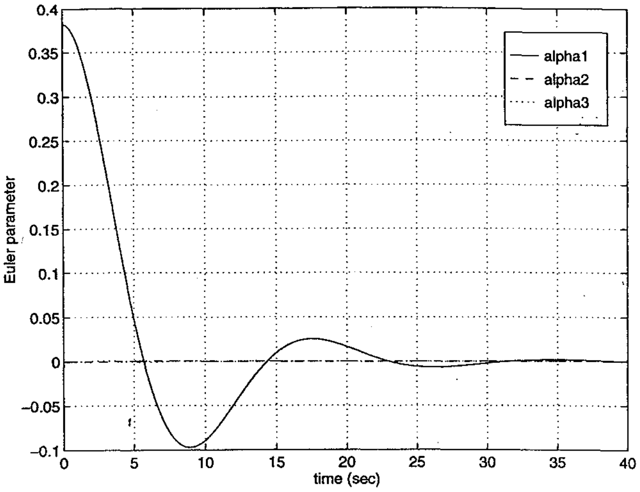

Fig. 5 Euler parameters.

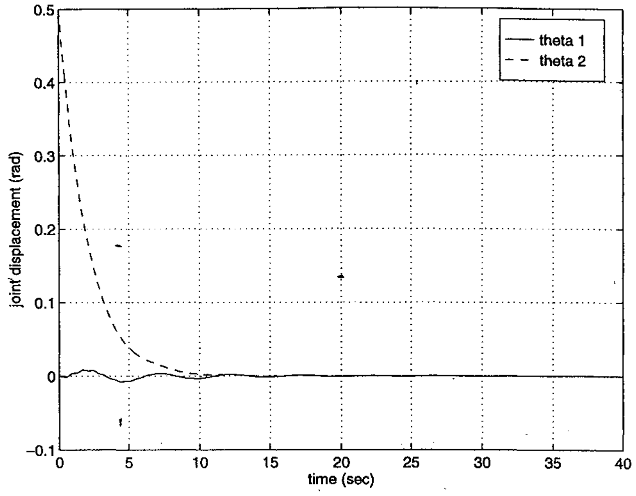

Fig. 6 Revolute joint displacements.

and each link is a 3-m-long flexible beam. The cross section of the links is circular with 1.0 cm diameter resulting in significant flexibility. The material chosen for the central body as well as the links has a mass density of 2.568×10^{-3} kg/m^3 and modulus of elasticity $E = 6.34 \times 10^9$ kg/m^2. The mass of the central body is 4030 kg and that of each link is 0.605 kg. The principal moments of inertia of the central body about local x, y, and z axes are 1600, 1600, and 500 kg-m^2, respectively. Each link can rotate about its local z axis. The moment of inertia of each link about its axis of rotation is 1.815 kg-m^2. The central body has three rotational degrees of freedom. As shown in Fig. 4, there are two revolute joints, one between the central body and link 1 and another between link 1 and link 2. The axes of rotation for revolute joints 1 and 2 coincide with the local z axes of links 1 and 2, respectively. A three-axis torque actuator is assumed for the central body and one torque actuator is assumed for each of the revolute joints. Inertial attitude and rate sensors are collocated with the central-body torque actuators. Joint angle and rate sensors are collocated with the torque actuators at the revolute joints. The first and second link are modeled as flexible beams with pinned–pinned and pinned–free boundary conditions, respectively. The first four bending modes in the local xy plane were considered for each link, i.e., the system has five rigid rotational degrees of freedom and eight flexible degrees of freedom. The modal data were obtained using MSC/NASTRAN.[15] A complete nonlinear simulation was obtained using a commercially available software, DADS.[16]

A rest-to-rest maneuver was considered in order to demonstrate the control law. The initial configuration was equivalent to $\pi/4$ rad rotation of the entire spacecraft about the global x axis and 0.5 rad rotation of the revolute joint 2 about its local z axis. The objective of the control law was to restore the zero state of the

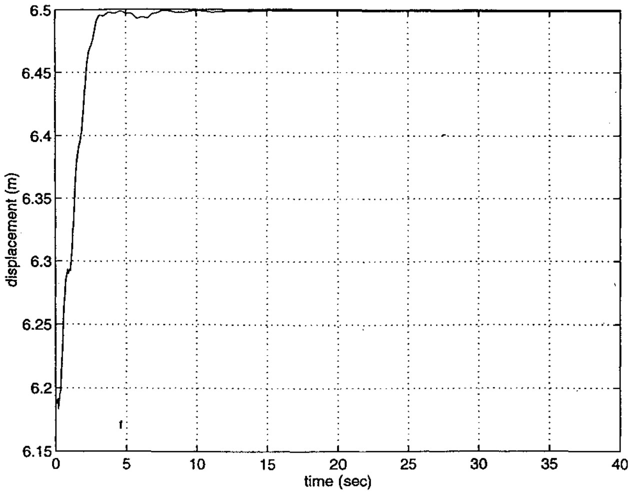

Fig. 7 *X* displacement of end effector.

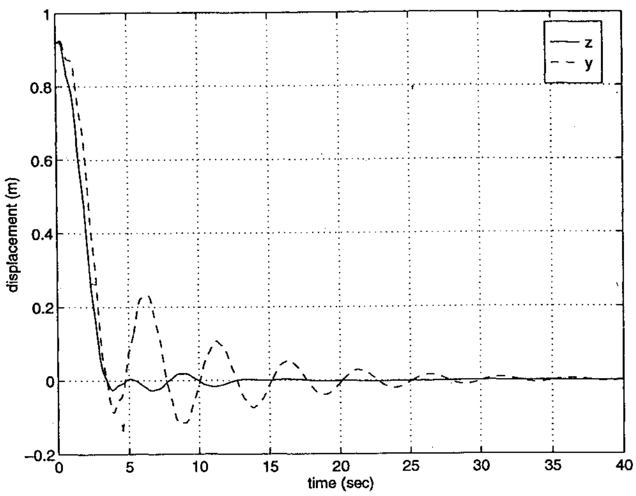

Fig. 8 *Y* and *Z* displacements of end effector.

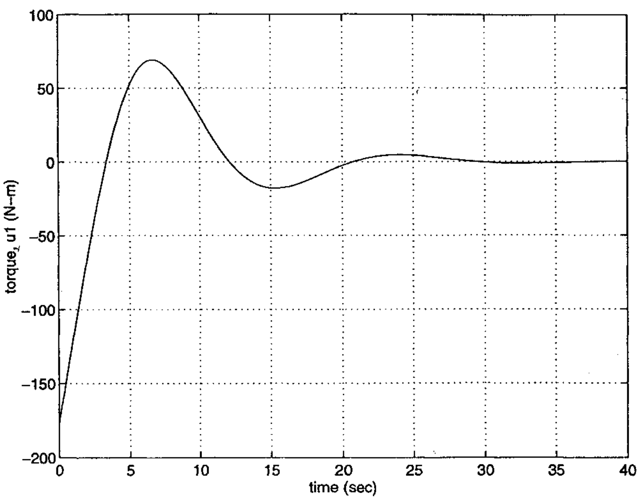

Fig. 9 Control torque for Euler axis 1.

system (i.e., zero attitude of the central body and fully stretched configuration of the links). The nonlinear dissipative control law of Eq. (13) was used to accomplish the task. Since there are no known techniques for the synthesis of such controllers, the selection of controller gains was based on trial and error. Based on several trials, the following gains were found to give the desirable response: $G_{p1} = \mathrm{diag}(1000, 1000, 1000)$, $G_{p2} = \mathrm{diag}(50, 50)$, and $G_r = \mathrm{diag}(500, 275, 270, 100, 100)$. As the system sets in motion,

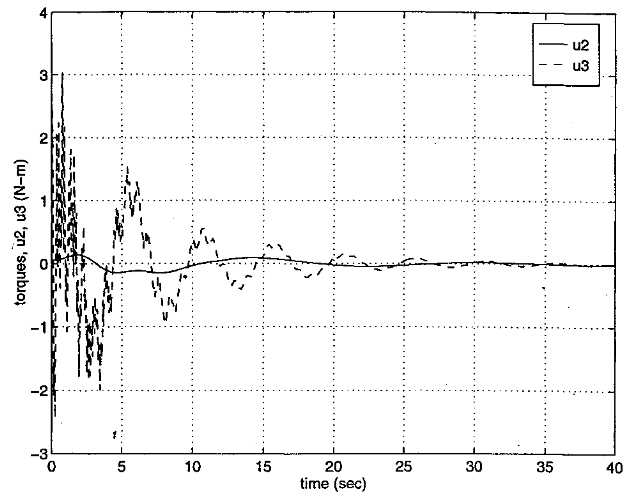

Fig. 10 Control torques for Euler axes 2 and 3.

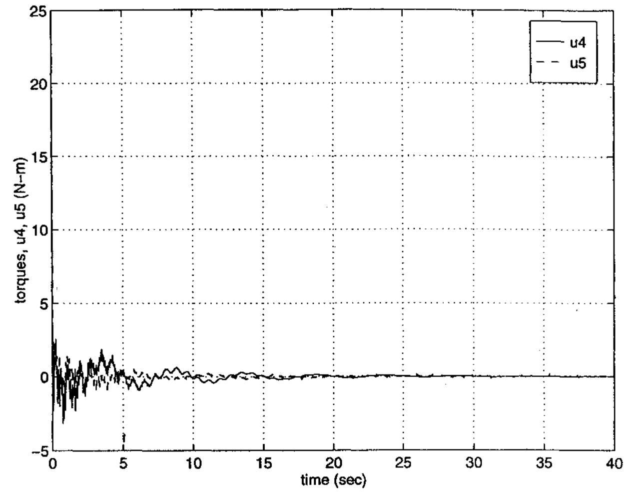

Fig. 11 Control torques for revolute joints 1 and 2.

all members move relative to one another and there is dynamic interaction between the members. Complete nonlinear effects and coupling effects are incorporated in the simulation. The Euler parameter responses are shown in Fig. 5. The term α_1 reaches steady state in about 30 s, and α_2, α_3 remain very small (less than 10^{-3}) during the maneuver. The joint angle displacements for revolute joints 1 and 2 are shown in Fig. 6. The joint displacements decay asymptotically and are nearly zero within 15 s. The end-effector displacements with respect to global x, y, and z axes are shown in Figs. 7 and 8. It can be seen that the x position of the end effector reaches its desired value in about 15 s, whereas the y and z positions take about 35 s to settle. These responses effectively demonstrate the stability result of Sec. III. The time histories of control torques are given in Figs. 9–11. The effects of nonlinearities in the model can be seen in the responses as well as in the torque profiles.

VI. Concluding Remarks

Stability of a class of nonlinear multibody flexible space systems was considered using a class of dissipative control laws. Assuming collocated actuators and sensors, global asymptotic stability was proved using a nonlinear feedback of the central-body quaternion angles, relative body angles, and angular velocities. A numerical example was also given to demonstrate the stability result. In addition, for the special case wherein the central-body motion is in the linear range whereas the appendages can undergo unlimited (nonlinear) motion, global asymptotic stability was proved with a linear static dissipative control law. Furthermore, the stability was shown to hold despite the presence of a broad class of actuator and sensor nonlinearities and actuator dynamics. A class of linear dynamic dissipative controllers was also introduced and was shown to provide global asymptotic stability. Dynamic dissipative controllers offer more design freedom than the static dissipative controller and therefore can potentially achieve better performance and noise attenuation. All the stability results presented depend only on inherent passivity characteristics of the system and are valid in spite of unmodeled modes and parametric uncertainties, i.e., the stability is robust to model errors. The results have significant practical value since mathematical models of multibody flexible systems usually have substantial inaccuracies and the actuation and sensing devices have nonlinearities.

Design of dissipative controllers to obtain optimal performance is as yet an unsolved problem. Future work should address the development of systematic methods for the synthesis of both nonlinear and linear as well as static and dynamic dissipative controllers for such systems.

Appendix: System Properties

Lemma A1. For the system represented by Eq. (2), the matrix $(1/2)\dot{M} - C$ is skew symmetric.

Outline of proof: Using the indicial notation, the (k, j)th element of $C(p, \dot{p})$ is defined as

$$c_{kj} = \sum_{i=1}^{n} c_{ijk}(p)\dot{p}_i = \sum_{i=1}^{n} \frac{1}{2}\left(\frac{\partial M_{kj}}{\partial p_i} + \frac{\partial M_{ki}}{\partial p_j} - \frac{\partial M_{ij}}{\partial p_k}\right)\dot{p}_i \quad \text{(A1)}$$

Similarly, the k_jth component of the time derivative of the inertia matrix $\dot{M}(p)$ is given by the chain rule as

$$\dot{M}_{kj} = \sum_{i=1}^{n} \frac{\partial M_{kj}}{\partial p_i}\dot{p}_i \quad \text{(A2)}$$

Now if we define the matrix $S = (1/2)\dot{M} - C$, then the (k, j)th element of S is given by

$$\begin{aligned} S_{kj} &= \frac{1}{2}\dot{M}_{kj} - C_{kj} \\ &= \frac{1}{2}\sum_{i=1}^{n}\left[\frac{\partial M_{kj}}{\partial p_i} - \left(\frac{\partial M_{kj}}{\partial p_i} + \frac{\partial M_{ki}}{\partial p_j} - \frac{\partial M_{ij}}{\partial p_k}\right)\right]\dot{p}_i \\ &= \frac{1}{2}\sum_{i=1}^{n}\left(\frac{\partial M_{ij}}{\partial p_k} - \frac{\partial M_{ki}}{\partial p_j}\right)\dot{p}_i \end{aligned} \quad \text{(A3)}$$

Since the inertia matrix is symmetric, i.e., $M_{ij} = M_{ji}$, by interchanging the indices k and j, it follows from the above equation that

$$S_{jk} = -S_{kj} \quad \text{(A4)}$$

which implies that the matrix S is skew symmetric. □

Lemma A1 can be used to prove that the system given by Eq. (2) has the important inherent property of passivity, as defined in Ref. 3.

Lemma A2. The input–output map from u to y_r is passive; i.e. (with zero initial conditions),

$$\int_0^T y_r^T(t)u(t) \geq 0 \qquad \forall T \geq 0 \quad \text{(A5)}$$

for all $u(t)$ belonging to the extended Lebesgue space L_{2e}^k.

Proof. Premultiplying both sides of Eq. (2) by $\dot{p}^T$ and integrating, we get

$$\int_0^T [\dot{p}^T M(p)\ddot{p} + \dot{p}^T C(p, \dot{p})\dot{p} + \dot{p}^T D\dot{p} + \dot{p}^T Kp] = \int_0^T y_r^T u \,\mathrm{d}t \quad \text{(A6)}$$

Noting that

$$\frac{\mathrm{d}}{\mathrm{d}t}[\dot{p}^T M(p)\dot{p}] = 2\dot{p}^T M(p)\ddot{p} + \dot{p}^T\dot{M}(p)\dot{p} \quad \text{(A7)}$$

and using Lemma A1, we get

$$\frac{1}{2}\dot{p}^T(T)M(p(T))\dot{p}(T) + \int_0^T \dot{p}^T D\dot{p}\,\mathrm{d}t + \frac{1}{2}p^T(T)Kp(T) = \int_0^T y_r^T u\,\mathrm{d}t \quad \text{(A8)}$$

which gives the required result. □

References

[1]Desoer, C. A., and Vidyasagar, M., *Feedback Systems: Input-Output Properties*, Academic, New York, 1975.

[2]Joshi, S. M., *Control of Large Flexible Space Structures*, Vol. 131, Lecture Notes in Control and Information Sciences, Springer–Verlag, Berlin, 1989.

[3]Spong, M. W., and Vidyasagar, M., *Robot Dynamics and Control*, Wiley, New York, 1989.

[4]Juang, J.-N., Wu, S.-C., Phan, M., and Longman, R. W., "Passive Dynamic Controllers for Nonlinear Mechanical Systems," *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 5, 1993, pp. 845–851.

[5]Kelkar, A. G., Joshi, S. M., and Alberts, T. E., "Passivity-Based Control of Nonlinear Flexible Multibody Systems," *IEEE Transactions on Automatic Control*, Vol. 40, No. 5, 1995, pp. 910–914.

[6]Kelkar, A. G., "Mathematical Modeling of a Class of Multibody Flexible Space Structures," NASA TM-109166, Dec. 1994.

[7]Haug, E. G., *Computer-Aided Kinematics and Dynamics of Mechanical Systems*, Allyn and Bacon Series in Engineering, Allyn and Bacon, Needham Heights, MA, 1989.

[8]Benhabib, R. J., Iwens, R. P., and Jackson, R. L., "Stability of Large Space Structure Control Systems Using Positivity Concepts," *Journal of Guidance and Control*, Vol. 4, No. 5, 1981, pp. 487–494.

[9]McLaren, M. D., and Slater, G. L., "Robust Multivariables Control of Large Space Structures Using Positivity," *Journal of Guidance, Control, and Dynamics*, Vol. 10, No. 4, 1987, pp. 393–400.

[10]Joshi, S. M., Maghami, P. G., and Kelkar, A. G., "Design of Dynamic Dissipative Compensators for Flexible Space Structures," *IEEE Transactions on Aerospace and Electronic Systems* (to be published).

[11]Anderson, B. D. O., "A System Theory Criterion for Positive-Real Matrices," *SIAM Journal of Control*, Vol. 5, 1967, pp. 171–183.

[12]Lozano-Leal, R., and Joshi, S. M., "Strictly Positive Real Functions Revisited," *IEEE Transactions on Automatic Control*, Vol. 35, No. 11, 1990, pp. 1243–1245.

[13]Joshi, S. M., and Gupta, S., "Robust Stabilization of Marginally Stable Positive-Real Systems," NASA TM-109136, July 1994.

[14]Van Valkenberg, M. E., *Introduction to Modern Network Synthesis*, Wiley, New York, 1965.

[15]Anon., "MSC/Nastran, Software for Finite Element Analysis by MacNeal-Schwendler Corporation," MacNeal-Schwendler Corp., Los Angeles, CA, 1992.

[16]Anon., "DADS User's Manual," Computer Aided Design Software, Inc., Oakdale, IA, July 1989.